Decision Analysis

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EXECUTIVE SUMMARY

- Decision analysis is a new technique that comprises a philosophy, theory, methodology, and professional practice necessary to formalize the analysis of decisions. It is a systems engineering-decision theory approach to decision-making, whereby the decision to be made forms the focus for the analysis. Decision analysis provides not only a philosophy and language for approaching decision problems, but also a logical and quantitative procedure for analyzing them. This report is intended to describe and illuminate the theory and practice of decision analysis.

- The decision analysis procedure is formally applied only to those problems that can justify careful analysis. These problems are characterized by several of the following elements:

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<th>Characteristics</th>
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<td>Uniqueness</td>
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<td>Long run implications</td>
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<td>Complex preferences</td>
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  - The problem is one of a kind.
  - A significant portion of the organization's resources is involved.
  - Many key factors are known only imperfectly.
  - The organization will be affected by the results of the decision for many years.
  - The desires of the decision-maker are not clearly formulated.

- The decision analysis procedure has been applied to a variety of decision situations: in the commercial area, to the introduction of a new product or the change in design of an old one; in the military area, to the acquisition of a new weapon or the best defense against that of a potential enemy; in the medical area, to the selection of a medical or surgical procedure for a patient; in the social area, to the regulation and operation of public utilities; and finally, in the personal area, to the selection of a new car, home or career. It has application to any decision susceptible to logical analysis, and is becoming practical in a growing number of fields.

- Decision analysis has already been successfully applied to decision problems in widely divergent areas. Two major applications—to problems of new product introduction and space program planning—are described in this report. In the future, decision analysis should show major growth in both its scope of applications and its effects on organizational procedures.

- The strength of decision analysis lies in increasing the logical quality of decision-making. This advantage will be important to managers who must present evidence of carefully reasoned and documented decisions. However, they will have to overcome the objections of those who feel that a problem is avoided rather than illuminated when it is placed in quantitative form.
INTRODUCTION

Decision analysis is a term used to describe a body of knowledge and professional practice for the logical illumination of decision problems. It is the latest link in a long chain of quantitative advances in management that have emerged from the operations research/management science heritage. It is the result of combining aspects of systems analysis and statistical decision theory. Systems analysis grew as a branch of engineering whose strength was consideration of the interactions and dynamic behavior of complex situations. Statistical decision theory was concerned with how to be logical in simple uncertain situations. When their concepts are merged, they can reveal how to be logical in complex, dynamic, and uncertain situations; this is the province of decision analysis.

Thus, decision analysis focuses logical power to reduce confusing and worrisome problems to their elemental form. It does this not only by capturing structure, but by providing conceptual and practical methods for measuring and using whatever knowledge regarding uncertainty is available, no matter how vague. When all available knowledge has been applied, the problem is reduced to one of preference; thus the best alternative will depend on the desires of the decision-maker. Here again, decision analysis provides conceptual and practical methods for measuring preferences. The problem may require expressing the relative desirability of various outcomes, the effect on desirability of changes in timing, and the tolerance for uncertainty in receiving outcomes. In particular, the impact of uncertainty upon the decision can be measured and interpreted—not left to intuition.

BACKGROUND

History of Quantitative Decision-Making

Operations Research

Operations research was the first organized activity in the scientific analysis of decision-making. It originated in the application of scientific methods to the study of air defense during the Battle of Britain. The development of operations research continued in the U.S. in the Navy’s study of antisubmarine and fleet protection problems. After World War II, many of the scientists experienced in operations research decided to apply their new tools to the problems of management.

However, an examination of the transition of operations research from military to civilian problems shows that the limitations inherent in the military applications carried over to the civilian work. Many of the operations researchers trained in the military environment had become used to working only on operationally repetitive problems. In these constantly recurring problems, the impact of the formal analysis became evident to even the most skeptical observers. Some of the researchers, however, concluded that only this type of problem was susceptible to scientific analysis—that is, they limited operations research to the study of repetitive processes.

Since repetitive decisions are also important to the civilian world, operations research made substantial headway in its new environment. Yet, the insistence on repetition confined the efforts of operations researchers within the province of lower and middle management, such as inventory control, production scheduling, and tactical marketing. Seldom did the analysts study decision problems relevant to the top executive.
Management Science

In the mid-1950s, operations research spawned an offshoot—management science. This discipline developed in response to a deep concern that the special problems of management were not receiving sufficient attention in operations research circles. This new field grew to emphasize science more than management, however. Management scientists have been accused of having more interest in those problems that are subject to elegant mathematical treatment than in those of the top executive, which are generally less easily quantified.

Although many students of business have considered the problems of top management, they have not generally had the scientific and mathematical training necessary to give substance to their ideas and to allow their application in new situations. When the top manager sought help on a problem, he often had to choose between a mathematician who was more concerned with the idiosyncrasies of the situation than with its essence and an experienced "expert" who might be tempted to apply an old solution to a radically new problem. Thus, the early promise of scientific aids for the executive was slow in materializing.

Decision Analysis

In the last few years, a new discipline, called "decision analysis," has developed from these predecessors. It seeks to apply logical, mathematical, and scientific procedures to the decision problems of top management that are characterized by the following:

- *Uniqueness.* Each is one of a kind, perhaps similar to—but never identical with—previous situations.
- *Importance.* A significant portion of the organization's resources is in question.
- *Uncertainty.* Many of the key factors that must be taken into account are imperfectly known.
- *Long run implications.* The enterprise will be forced to live with the results of the situation for many years, perhaps even beyond the lifetimes of all individuals involved.
- *Complex preferences.* The task of incorporating the decision-maker's preferences about time and risk assumes great importance.

Decision analysis provides a logical framework for balancing all these considerations. It permits mathematical modeling of the decision, computational implementation of the model, and quantitative evaluation of the various courses of action. This report describes and delineates the potential of decision analysis as an aid to top management.

The Timeliness of Decision Analysis

An appropriate question is why decision analysis has only recently emerged as a discipline capable of treating the complexities of significant decision problems. The answer is found in the combination of three factors: historical circumstance, development of complementary capabilities, and the need for increased formalism.

The Computer Revolution

Despite the elaborateness of its logical foundations, decision analysis would be merely an intellectual curiosity rather than a powerful tool if the means were not available to build models and to manipulate them economically. The rapid development of the electronic computer in the past two decades has made feasible what would have been impossible only a quarter of a century ago. The availability of electronic computation is an essential condition for the growth of the decision analysis field.

The Tyranny of the Computer

A powerful tool is always subject to misuse. The widespread use of computers has led some managers to feel that they are losing rather than gaining control over the operations of their organizations. These feelings can lead to a defensive attitude toward the sug-
gestion that computers should be included in the decision-making process.

Decision analysis can play a major role in providing the focus that management requires to control application of computers to management activities. When examined through decision analysis, the problem is not one of management information systems, but one of providing management with structured decision alternatives in which management experience, judgment, and preference have already been incorporated. Since properly applied decision analysis produces insight as well as answers, it places control in, rather than out of, the hands of the decision-maker.

The Need for Formalism

A final force in the current development of decision analysis is the trend toward professional management in present organizations. The one-man show is giving way to committees and boards, and the individual entrepreneur is becoming relatively less important. A concomitant of this change is the need for new professional managers to present evidence of more carefully reasoned and documented decisions. Even the good intuitive decision-maker will have to convince others of the logic of his decisions.

However, the need for more formalism may also be imposed from outside the organization. The nature of competition will mean that when one company in an industry capitalizes on the efficacy of decision analysis, the others will be under pressure to become more orderly in their own decision-making. To an increasing extent, good outcomes resulting from intuitive decisions will be regarded in the same light as winnings at the races—that is, as the result of luck rather than of prudent managerial practice.

The Essence of Decision Analysis

Definition of Decision

In describing decision analysis, the first step is to define a decision. In this report, a decision is considered an irrevocable allocation of resources, in the sense that it would take additional resources, perhaps prohibitive in amount, to change the allocation. Some decisions are inherently irrevocable, such as whether or not to amputate a pianist’s hand; others are essentially irrevocable, such as the decision by a major company to enter a new field of endeavor.

Clearly, no one can make a decision unless he has resources to allocate. For example, a manufacturer may be concerned about whether his competition will cut prices, but unless he can change something about the way he does business, he has no decisions to make. Concern without the ability to make decisions is simply “worry.” It is not unusual in practice to encounter decision problems that are really worries. Exposing a decision problem as a worry may be very helpful if it allows the resources of the decision-maker to be devoted more profitably to other concerns.

Another common phenomenon is the study, which is an investigation that does not focus on a decision. Until a decision must be made, how can the economic balance of the study be determined? For example, suppose someone requested a study of the automobile in his particular community. The person conducting the study might survey cars’ weight, horsepower, displacement, braking ability, seating capacity, make, type, color, age, origin, and on and on. However, if a decision were required concerning the size of stalls in a parking facility, or the length of a highway acceleration lane, the pertinent characteristics would become clear. Further, decision analysis could even determine how extensive a survey, if any, would be economic. Thus, concentrating on a decision to be made provides a direct focus to the analysis that is achievable in no other way. Studies, like worries, are not our concern: decisions are.

The next step is to define a decision-maker: an individual who has the power to commit the resources of the organization. In some cases, the decision-maker may be an organiza-
tional entity, such as an executive committee. It is important, however, to distinguish advisory individuals or bodies from those with the power to commit the organization. Study upon study may be performed within an organization advocating or decrying a certain course of action, but until resources are committed, no decision has been made. The first step in any decision analysis is the identification of the responsible party.

The Distinction Between a Good Decision and a Good Outcome

Before there can be a formal discussion of decision analysis, the distinction between a good decision and a good outcome must be understood. A good decision is one based on the information, values, and preferences of a decision-maker. A good outcome is one that is favorably regarded by a decision-maker. It is possible to have good decisions produce either good or bad outcomes. Most persons follow logical decision procedures because they believe that these procedures, speaking loosely, produce the best chance of obtaining good outcomes.

To illustrate this point, suppose that we had agreed to serve as decision analysis consultants to a person who said that he would engage only in gambles that were weighted in his favor. Then this person informed us that he had purchased a ticket in a lottery. There were 100 tickets in the lottery, the prize was $100, and he paid $10 for the ticket. We demonstrate to him that with 1 chance in 100 of winning the $100, his expected income from the ticket is only 1/100 of $100 or $1, so that having paid $10 for the ticket, his expected loss on the entire prospect is $9. Consequently, in view of this person's expressed desire to avoid unfavorable gambles, we say that he has made a bad decision.

However, the next day he receives a check for $100 as a consequence of having won the lottery; everyone agrees that this is a good outcome for him. Yet we must report that his decision was bad in spite of the good outcome, or, perhaps better, that his outcome was good in spite of the bad decision. This would be a proper situation to be described as “lucky.”

Suppose, however, that the person had paid only 10 cents for his ticket. In this case, his expected income is still $1, but because he spent only 10 cents for the ticket, his net expected earnings are 90 cents. Consequently, we would compliment him on his good decision. Yet if no winnings check appears on the next day, the client has now experienced a bad outcome from his good decision.

The distinction between good outcomes and good decisions is especially important in maintaining a detached, professional attitude toward decision problems. Recriminations based on hindsight in the form of “Why didn’t it work?” are pointless unless they reveal that available information was not used, that logic was faulty, or that the preferences of the decision-maker were not properly encoded. The proper framework for discussing the quality of decisions and outcomes is a major aid in using hindsight effectively.

Decision Analysis as a Language and a Philosophy

The decision analysis formalism serves both as a language for describing decision problems and as a philosophical guide to their solution. The existence of the language permits precision in specifying the many factors that influence a decision.

The most important feature of the language is its ability to represent the uncertainty that inevitably permeates a decision problem. The language of probability theory is used with only minor changes in terminology that reflect a subjective interpretation of probabilistic measurement. We regard probability as a state of mind rather than of things. The operational justification for this interpretation can be as simple as noting the changing odds on a sporting contest posted by gamblers as information about the event changes. As new information arrives, a new probability assignment is made. Decision analysis uses the
same subjective view of probability. By so doing, statements regarding uncertainty can be much more precise. Rather than saying, “There is some chance that a bad result is likely,” or an equivalent ambiguous statement, we shall be able to speak directly of the probability of a bad result. There is no need for vagueness in the language that describes uncertainty. Putting what is not known on the record is the first step to new knowledge.

Decision analysis can also make a major contribution to the understanding of decision problems by providing a language and philosophy for treating values and preferences. “Values” mean the desirability of each outcome; “preferences” refer to the attitudes of the decision-maker toward postponement or uncertainty in the outcomes he receives. Placing values and preferences in unambiguous terms is as unusual in current decision-making as is the use of direct probability assignments. Yet both must be done if the procedure is to be used to full advantage.

Later sections of this report describe the theory and practice of assigning probabilities, values, and preferences, but the impact of thinking in such terms can be indicated here. A most important consequence of formal thought is the spontaneous resolution of individual differences that often occur when the protagonists can deal in unambiguous terms. Two people who differ over the best alternative may find their disagreements in the areas of probability assignment, value, or preference. Thus, two men who are equally willing to take a risk may disagree because they assign different probabilities to various outcomes; or two men who assign the same probability to the outcomes may differ in their aversion to risk. It is unlikely that the nature of the disagreement will emerge without the formal language. More likely, epithets such as “foolhardy” or “rock-bound conservative,” will prevent any communication at all.

The decision analyst must play a detached role in illuminating the decision problem if he is to resolve differences. He must be impartial, never committing himself to any alternative, but rather showing how new information or changes in preference affect the desirability of available alternatives. The effectiveness of the decision analyst depends as much on his emotional detachment as on his knowledge of formal tools.

Decision analysis is a normative, rather than a descriptive, approach to decision problems. The decision analyst is not particularly interested in describing how decision-makers currently make decisions; rather he is trying to show how a person subscribing to certain logical rules would make these decisions in order to maximize attainment of his objectives. The decision procedures are derived from logic and from the desires of the decision-maker and are in this sense prescriptive.

Decision analysis is more than a language and a philosophy, but the experience of its users justifies it on this basis alone. By focusing on central issues, the approach often illuminates the best course of action in a way that makes discord evaporate.

**Decision Analysis as a Logical and Quantitative Procedure**

Decision analysis provides not only the philosophical foundations, but also a logical and quantitative procedure for decision-making. Since decision analysis encodes information, values, and preferences numerically, it permits quantitative evaluation of the various courses of action. Further, it documents the state of information at any stage of the problem and determines whether the gathering of further information is economically justifiable. The actual implementation of decision analysis models is typically a computer program that enables the many facets of the problem to be examined together. Most of this report will describe how the philosophy of decision analysis carries over into practice.

**Delegation of Responsibility**

Decision analysis provides both philosophical and operational guidelines for delegating
responsibility in an organization. If we want someone to make a good decision, we must provide that individual not only with the information but also with the values and preferences that are relevant to the decision. The key principle is that the delegator must supply a subordinate decision-maker with whatever information, values, and preferences required for him to reach the same decision that the delegating individual would have reached in the same situation. While few organizations currently use decision analysis principles in handling the problem of delegation, these principles are available when needed. It is rare that an organization performs a decision analysis on one of its major decisions without simultaneously obtaining new insight into its organizational structure.

THE DECISION ANALYSIS CYCLE

Decision analysis as a procedure for analyzing a decision is described below. This procedure is not an inviolable method of attacking the problem, but is a means of ensuring that essential steps have been consciously considered.

The figure describes decision analysis in the broadest terms. The procedure is iterative and comprises three phases. The first is a deterministic phase, in which the variables affecting the decision are defined and related, values are assigned, and the importance of the variables is measured without any consideration of uncertainty.

The second, or probabilistic, phase introduces probability assignments on the important variables and derives associated probability assignments on values. This phase also introduces the assignment of risk preference, which provides the best solution in the face of uncertainty.

The third, or informational, phase reviews the results of the last two phases to determine the economic value of eliminating uncertainty in each of the important variables in the problem. In some ways, this is the most important phase because it shows just what it could cost in dollars and cents not to have perfect information. A comparison of the value of information with its cost determines whether additional information should be collected.

If there are profitable further sources of information, then the decision should be to gather the information rather than to make the primary decision at this time. Thereupon will follow the design and execution of the information-gathering program, whether it be a market survey, a laboratory test, or military field trials.

The information that results from this program may change the model and the probability assignments on important variables. Therefore, the original three phases must be performed once more. However, the additional work required to incorporate the modifications should be slight and the evaluation rapid. At the decision point, it may again be profitable to gather new information and repeat the cycle or it may be more advisable to act. Eventually, the value of new analysis and information-gathering will be less than its cost, and the decision to act will then be made.

This procedure will apply to a variety of decision situations: in the commercial area, to the introduction of a new product or the change in design of an old one; in the military area, to the acquisition of a new weapon or the best defense against that of a potential enemy; in the medical area, to the selection of a med-
ical or surgical procedure for a patient; in the social area, to the regulation and operation of public utilities; and finally, in the personal area to selection of a new car, home or career. In short, the procedure can be applied to any decision susceptible to logical analysis.

The Deterministic Phase

Descriptions of the various phases of the procedure follow beginning with the deterministic phase. The deterministic phase is essentially a systems analysis of the problem. Within this phase, efforts devoted to modeling are distinguished from efforts devoted to analysis. The elements of the phase appear in Figure 2.

![Diagram of the Deterministic Phase](image)

**Fig. 2**
The Deterministic Phase

**MODELING:**
- Bound Decision
- Identify Alternatives
- Establish Outcomes
- Select System Variables
- Create Structural Model
- Create Value Model
- Create Time Preference Model

**ANALYSIS:**
- Measure Sensitivity
  - to Decision Variables
  - to State Variables

Modeling

Modeling is the process of representing the various relationships of the problem in formal, mathematical terms. The first step in modeling is to bound the decision, to specify precisely just what decision must be made. This requires listing in detail the perceived alternatives. Identification of the alternatives will separate an actual decision problem from a worry.

The next step—finding new alternatives—is the most creative part of decision analysis. New alternatives can spring from radically new concepts; more often they may be careful combinations of existing alternatives. Discovering a new alternative can never make the problem less attractive to the decision-maker; it can only enhance it or leave it unchanged. Often the difficulty of a decision problem disappears when a new alternative is generated.

The next step is to specify the various outcomes that the set of alternatives could produce. These outcomes are the subsequent events that will determine the ultimate desirability of the whole issue. In a new product introduction, for example, the outcomes might be specified by sales levels and costs of production or even more simply by yearly profits. Thus, there is a certain amount of arbitrariness in what to call an outcome. For decision analysis, however, an outcome is whatever the decision-maker would like to know in retrospect to determine how the problem came out. In a military problem, the outcome could be a complicated list of casualties, destruction, and armament expenditures; in a medical problem, it could be as simple as whether or not the patient dies.

Now comes the challenging process of selecting the system variables for the analysis, which are all those variables on which the outcomes depend. We can identify the system variables by imagining that we have a crystal ball that will answer any numerical questions relative to the decision problem, except, of course, which alternative to select. We could ask it questions about the outcome variables directly, thereby making them the only system variables in the problem. But typically outcome variables are difficult to think about in advance in the real world, and so we might choose to relate the outcome variables to others that are easier to comprehend. For
example, we might like to know the sales level of a new product. Or in lieu of this, we might attempt to relate the sales to our own price and quality and the competitors’ price and quality, factors that we might regard as more accessible. These factors would then become system variables in the analysis.

The selection of system variables is therefore a process of successive refinement, wherein the generation of new system variables is curtailed by considering the importance of the problem and the contributions of the variables. Clearly, allocation of the national budget can economically justify the use of many more system variables than can the selection of a new car.

Once we have decided on the system variables to use in the problem, each one must be distinguished either as a variable under the decision-maker’s control or as a variable determined by the environment of the problem. System variables that are under the decision-maker’s control are called decision variables. The selection of an alternative in a decision problem is really the specification of the setting of the decision variables. For example, in the new product introduction problem, the product price and the size of production facilities would both be decision variables.

System variables in the problem that are determined by the environment are known as state variables. Although state variables may have a drastic effect on the outcomes, they are autonomous, beyond the control of the decision-maker. For example, in the new product introduction, the cost of a crucial raw material or the competitor’s advertising level might be state variables.

We shall want to examine the effect of fluctuations in all system variables, whether decision variables or state variables. To aid in this task, the decision-maker or his surrogate must specify for each system variable a nominal value and a range of values that the variable may take on. In the case of a decision variable, the nominal value and range are determined by the decision-maker’s preconception regarding the interesting alternatives. In the case of state variables, the nominal value and range reflect the uncertainty assigned to the variables. For convenience, we can often think of the nominal value of a state variable as its expected value in the mathematical sense and of the range as the 10th percentile and 90th percentile points of its probability distribution.

Selecting system variables and setting nominal values and ranges require extensive consultation between the decision-maker and the decision analyst. At this stage, it is better to err by including a variable that will later prove to be unimportant than it is to eliminate a variable prematurely.

The next step is to specify the relationships among the system variables. This is the heart of the modeling process—i.e., creating a structural model that captures the essential interdependencies of the problem. This model should be expressed in the language of logic—mathematics—typically by a set of equations relating the system variables. In most decisions of professional interest, these equations will form the basis for a computer program to represent the model. The program provides rapid evaluation of model characteristics at modest cost.

Constructing a model of this type requires a certain sophistication in the process of orderly description and a facility for careful simplification. The procedure is elementary, but not trivial; straightforward, but not pedestrian.

Now the decision-maker must assign values to outcomes. Just as there was difficulty in defining an outcome, so there may be some question about the distinction between an outcome and its value. For example, in a business problem, the decision-maker may think of his future profit as both the outcome and the value associated with it. However, maintaining the generality of the formulation requires creating a distinction between the two.

To illustrate the necessity for this, consider a medical question involving the amputation
of an arm. The outcomes of interest might be complete recovery, partial recovery, or death, each with or without the operation. These outcomes would describe the results but would not reveal their value. For example, if the patient were a lawyer, he might consider death by far the most serious outcome and be willing to undergo the amputation if it sufficiently reduced the probability of death. These feelings might be based on the observation that an arm is not essential to his career. To a concert pianist, however, amputation might be worse than death itself, since life without being able to play might be unbearable. Consequently, he would be rational in refusing the amputation even if this choice made his death more likely.

Although in some cases the decision can be reached as a result of ordering outcomes in terms of desirability, most problems of practical interest require a numerical (cardinal) ranking system. Therefore, assigning a value means assigning a numerical value to an outcome. Though there may be many elements of value in the outcome, the final value assignment is a single number associated with that outcome.

In commercial situations, the value assigned to an outcome will typically be some form of profit. In social and military problems, however, the value assignment is more difficult because it requires measuring the value of a human life, or a cultured life, or a healthy life in dollars and cents terms. Though these questions of evaluation may be difficult, logic demands that they be approached directly in monetary terms if monetary resources are to be allocated.

The final step in creating the deterministic model is to specify the time preference of the decision-maker. Time preference is the term used to describe the human phenomenon of impatience. Everyone wants good things to happen to him sooner rather than later. This impatience is reflected in a willingness to consume less now rather than postpone the consumption. The payment of interest on savings accounts and the collection of interest on loans are mere reflections of this phenomenon. Consequently, representing the desires of a decision-maker requires a realistic mechanism for describing his time preference, a mechanism that reduces any time stream of value to a single number called worth.

For a corporate financial decision, worth will often be simply the discounted difference between future income and expenditures using an interest rate that depends upon the relationship of the corporation to its financial environment. In the military or medical fields, worth may be more difficult to establish.

The modeling part of the deterministic phase thus progresses from the original statement of the decision problem to a formal description suitable for detailed examination by logical and computational analysis. The decision-maker’s value assignments and his time preference permit rating any outcome that appears as a time stream first as a set of values in time and then as an equivalent worth.

Analysis

Analysis based on the deterministic phase centers on observing how changes in the variables affect worth. Experimentation of this type is known as sensitivity analysis; it is highly effective in refining the formulation of the problem.

The first sensitivity analysis we perform is associated with the decision variables. First, fixing all other state variables in the problem at their nominal values, we then allow one of the decision variables to traverse its assigned range and observe how worth changes. Of course, these observations are usually carried out by computer program. If we find that a particular decision variable has a major effect, then we know that we were correct in including it in the original formulation. But if a decision variable has little or no effect, we are justified in considering its removal as a decision variable. If reflection reveals that the latter is the case, we would say that we have eliminated an impotent decision variable. For
example, the time of introduction of a new product might seem to be a decision variable of major importance, but because of the combined effects of competitive reaction and the gaining of production experience, it might turn out to have very little effect. The timing of entry would then be an impotent variable.

Next, we perform sensitivity analyses on the state variables, which are uncertain and over which the decision-maker has no control. With all other system variables at their nominal values, we observe the change in worth while sweeping one state variable over its range. If a state variable has a major effect, then the uncertainty in the variable deserves special attention. Such variables are called aleatory variables to emphasize their uncertainty.

If, however, varying a state variable over its range produces only a minor change in worth, then that variable might well be fixed at its nominal value. In this case, we say that the state variable has become a fixed variable. A state variable may become fixeded either because it has an important influence on the worth per unit of its range, but an extremely small range, or because it has little influence on the worth per unit of its range, even though it has a broad range.

There is no reason to conclude that a fixed variable is unimportant in an absolute sense. For example, the corporate tax rate may be a fixed variable in a problem because no change in it is anticipated within the time period under consideration. Yet it is possible that an unforeseen large change in this rate could change a favorable venture into an unfavorable one.

Although sensitivity analysis has been described as if it concerns only changes in one variable at a time, some of the most interesting sensitivity results are often observed when there are simultaneous changes in state variables. Since the possibilities of changing state variables jointly grows rapidly with the number of state variables, an important matter of judgment for the decision analyst is to determine the amount of simultaneous sensitivity analysis that is economic.

**The Probabilistic Phase**

The net result of the deterministic sensitivity analysis on the autonomous state variables is to divide them into aleatory and fixed classes. The probabilistic phase determines the uncertainty in value and worth due to the aleatory variables. The phase will be divided into steps of modeling and analysis; Figure 3 illustrates its internal structure.

![Diagram](image)

**Fig. 3 The Probabilistic Phase**

- **MODELING:**
  - Encode Uncertainty on Aleatory Variables
  - Encode Risk Preference

- **ANALYSIS:**
  - Develop Worth Lotteries and Certainty Equivalent
  - Measure Stochastic Sensitivity
  - Measure Risk Sensitivity

**Modeling Probability Distributions**

The first modeling step in the probabilistic phase is the assignment of probability distributions to the aleatory variables. Either the decision-maker or someone he designates must assign the probability that each aleatory variable will exceed any given value. If any set of aleatory variables is dependent, in the sense that knowledge of one would provide information about the others, then the probability assignments on any one variable must
be conditional on the values of the others. Gathering these assignments amounts to asking such questions as, "What are the odds that sales will exceed 10 million units in the first year?" (See section entitled "Encoding Knowledge and Preferences.") Strange as such questions may be in the current business world, they could be the standard executive language of tomorrow.

Analysis

With knowledge from the deterministic phase of how the worth depends on the state variables and assigned probability distributions on the aleatory variables, it is a straightforward calculation to determine the probability distribution of worth for any setting of the decision variables; this probability distribution is the "worth lottery." The worth lottery describes the uncertainty in worth that results from the probability assignments to the aleatory variables for any given alternative (setting of decision variables.) Of course, the values of the fixated variables are never changed.

To select a course of action, the analyst could generate a worth lottery for each alternative and then select the one that is more desirable. But how would he know which worth lottery is most desirable to the decision-maker?

One important principle that allows judging one worth lottery as being better than another is that of stochastic dominance, which is illustrated in Figure 4. Part A of this figure shows the worth lottery for two alternatives in both probability densities and excess probability distribution forms. The excess probability distribution, or excess distribution, is the probability that the variable will exceed any given value plotted as a function of that value. Its height at any point is the area under the probability density function to the right of that point. Comparison of the excess distributions for the two alternatives reveals that, for any value of \( X \), there is a higher probability that alternative 2 will produce a worth in excess of that \( X \) than will alternative 1. Consequently, a decision-maker preferring more worth to less would prefer alternative 2. If alternative \( A \) has an excess distribution that is at least as great as that of alternative \( B \) at any point and greater than \( B \) at at least one point, alternative \( A \) stochastically dominates alternative \( B \). If stochastic dominance exists between two competing alternatives, there is no need to inquire into the risk preference of the decision-maker, who rationally must rule out the stochastically dominated alternatives.

Part B of Figure 4 illustrates a case in which stochastic dominance does not exist. The excess distributions on worth for the two alternatives cross. If the decision-maker wants to maximize his chance of receiving at
least a small amount of worth, he would prefer alternative 1; if he wants to maximize his chance of receiving at least a large amount of worth, he would prefer alternative 2. In situations like this, where stochastic dominance does not apply, the risk preference of the decision-maker must be encoded formally, as shown below.

Just because alternative $A$ stochastically dominates alternative $B$ does not mean that the decision-maker will necessarily achieve a higher worth by following alternative $A$. For example, if alternative $A$ produces worths of five to 15 with equal probability and alternative $B$ produces worths of zero and ten with equal probability, then $A$ stochastically dominates $B$. Yet it is possible that $A$ will produce a worth of five while $B$ will produce a worth of ten. However, not knowing how the lottery will turn out, the rational man would prefer alternative $A$.

**Modeling Risk Preference**

If stochastic dominance has not determined the best alternative, the analyst must turn to the question of risk preference. To demonstrate that most individuals are averse to risk, it is only necessary to note that few, if any, are willing to toss a coin, double or nothing, for a year’s salary. Organizations typically act in the same way. A realistic analysis of decisions requires capturing this aversion to risk in the formal model.

Fortunately, if the decision-maker agrees to a set of axioms about risk taking (to be described in the following section), his risk preference can be represented by a utility curve like that shown in Figure 5. This curve assigns a utility to any value of worth. As a consequence of the risk preference axioms, the decision-maker’s rating of any worth lottery can be computed by multiplying the utility of any possible worth in the lottery by the probability of that worth and then summing over all possible worths. This rating is called the expected utility of the worth lottery.

If one worth lottery has a higher expected utility than another, then it must be preferred by the decision-maker if he is to remain consistent with the axioms. The analyst is not telling the decision-maker which worth lottery he should prefer but only pointing out to him a way to be consistent with a very reasonable set of properties he would like his preferences to enjoy.

Thus, the utility curve provides a practical method of incorporating risk preference into the model. When faced with a choice between two alternatives whose worth lotteries do not exhibit stochastic dominance, the analyst computes the expected utility of each and chooses the one with the higher expected utility.

Although the expected utility rating does serve to make the choice between alternatives, its numerical value has no particular intuitive meaning. Therefore, after computing the expected utility of a worth lottery, the analyst often returns to the utility curve to see what worth corresponds to this expected utility; we call this quantity the certain equivalent worth of the worth lottery. The name arises as follows: if another worth lottery produced the certain equivalent worth with probability one, then it and the original lottery would have the same expected utilities and hence would be equally preferred by the decision-maker. Consequently, the certain equivalent worth of any worth lottery is the amount of worth received for certain, so that the decision-maker would be indifferent between receiving this worth and participating in the lottery. Since almost all utility curves show
that utility increases as worth increases, worth lotteries can be ranked in terms of their certain equivalent worths. The best alternative is the one whose worth lottery has the highest certain equivalent worth.

Analysis

In returning to the analysis of the probabilistic phase, the first step is to compute the certain equivalent worth of each of the alternatives. Since the best decision would be the alternative with the highest certain equivalent worth, the decision probably could be considered solved at this point. The careful analyst, however, will examine the properties of the model to establish its validity and so would not stop here. The introduction of risk preference is another point at which to check the sensitivity of the problem. For example, by setting all decision variables but one to their nominal values and then sweeping this one decision variable through its range, the analyst may find that although this variation changes the worth lottery it does not significantly change the certain equivalent worth. This result would indicate that the decision variable could be fixed at its nominal value.

Aleatory variables receive the same sensitivity analysis by setting one of them equal to a trial value within the range and then allowing the others to have the appropriate conditional joint probability distribution. When the decision variables are given their nominal values, the program will produce a worth lottery and hence a certain equivalent worth for the trial value. Sweeping the trial value from one end of its range to the other shows how much certain equivalent worth is changed. If the change is small, there is evidence that the particular aleatory variable may be changed to a fixed variable. We call this procedure measurement of the stochastic sensitivity of a variable. It is possible that an aleatory variable showing a large deterministic sensitivity could reveal only a small stochastic sensitivity and vice versa. Consequently, any decisions to remove variables from aleatory status on the basis of deterministic sensitivity might well be reviewed at this time by measurement of stochastic sensitivity.

As in the case of deterministic sensitivity, we can measure the stochastic sensitivity of many variables, simultaneously. Once more, the decision analyst must judge how far it is profitable to proceed. Measurement of stochastic sensitivity is a powerful tool for locating the important variables of the problem.

There is one other form of sensitivity analysis available at this point: risk sensitivity. In some cases, it is possible to characterize the utility curve by a single number—the risk aversion constant (just when this is possible will be discussed later). However, when the risk aversion constant is applicable we can interpret it as a direct measure of a decision-maker’s willingness to accept a risk. An individual with a small risk aversion constant is quite willing to engage in a fair gamble; he has a tolerant attitude toward risk. As his risk aversion constant increases, he becomes more and more unwilling to participate. If two men share responsibility for a decision problem, the less risk tolerant will assign a lower certain equivalent worth for any given worth lottery than will the other. Perhaps, however, when the certain equivalent worths are computed for all alternatives for both men, the ranking of certain equivalent worths might be the same for both, or at least the same alternative would appear at the top of both lists. Then there would hardly be any point in their arguing over the desirable extent of risk aversion and a possible source of controversy would have been eliminated.

The measurement of risk sensitivity determines how the certain equivalent worths of the most favorable alternatives depend on the risk aversion constant. The issue of risk aversion can often be quickly resolved.

The problem structure, the set of alternatives generated, the probability assignment to aleatory variables, the value assessments, the statement of time preference, and the specification of risk preference combine to indicate
the best alternative in the problem. The overall procedure is illustrated by the decision analysis pyramid in Figure 6. However, it still may be best to obtain more information rather than to act. This determination is made in the third phase, as described below.

The Informational Phase

The informational phase is devoted to finding out whether it is worthwhile to engage in a possibly expensive information-gathering activity before making a decision. It is, in the broadest sense, an experimental design procedure from which one very possible result is the decision to perform no experiment at all. Figure 7 shows the steps in the phase.
given the available information. The computer program would then determine the expected utility of the entire decision problem including the payment to the clairvoyant, all conditional on his reporting s.

Before engaging the clairvoyant, however, the probability to be assigned to his reporting s as the value of the particular aleatory variable is described by the probability distribution showing the current state of knowledge on this variable. Consequently, we obtain the expected utility of purchasing his information on the variable at a cost k by multiplying the expected utility of the information given that he reports s and costs k, by the current probability that he will report s and then summing over all values of s. The analyst uses the current probability in this calculation because if the clairvoyant is reliable, the chance of his reporting that the variable falls in any range is just the chance that it will fall in that range.

Knowing the expected utility of purchasing the information from the clairvoyant at a cost of k, we can gradually increase k from zero until the expected utility of purchasing the information is just equal to the expected utility of proceeding with the decision without clairvoyant information. The value of k that establishes this equivalence is the value of clairvoyance on the aleatory variable.

The value of clairvoyance on an aleatory variable represents an upper bound on the payment for any experimental program designed to provide information on this variable, for no such program could be worth more than clairvoyance. The actual existence of a clairvoyant is not material to this discussion; he is merely a construct to guide our thinking.

We call the process of measuring the value of clairvoyance the measurement of economic sensitivity. If any aleatory variable exhibits high economic sensitivity, it is a prime candidate for an information-gathering program. It is possible, however, for a variable to have a high stochastic sensitivity and a low economic sensitivity because the available alternatives cannot take advantage of the information received about the variable. To determine the importance of joint information, the analyst can measure the value of clairvoyance on more than one variable at a time.

The actual information-gathering programs available will seldom provide perfect information, so they will be less valuable than clairvoyance. Extension of the discussion of clairvoyance shows how their value can be measured. Whereas the clairvoyant reported a particular value s for an aleatory variable, a typical experimental program will provide only a new probability distribution for the aleatory variable. The analyst would then determine the best decision, given this new information, and compute the expected utility of the decision problem. He would next multiply the expected utility by the probability that the experimental program would come out in this way and then sum over all possible outcomes of the experimental program. The result would be the expected utility of the experimental program at a given cost. The cost that would make the expected utility just equal to the expected utility of the problem without the experimental program would be the value of the experimental program. If the value is positive, it represents the maximum that one should pay for the program. If the value is negative, it means that the experimental program is expected to be unprofitable. Consequently, even though it would provide useful information, it would not be conducted.

Modeling

At this stage, the decision-maker and the analyst must identify the relevant information-gathering alternatives, from surveys to laboratory programs, and find which, if any, are expected to make a profitable contribution to the decision problem. In considering alternatives, they must take into account any deleterious effect of delay in making the primary decision. When the preferred information-gathering program is performed, it will lead, at least, to new probability assignments.
on the aleatory variables; it might also result in changing the basic structure of the model. When all changes that have been implied by the outcome of the experimental program are incorporated into the model, the deterministic and probabilistic phases are repeated to check sensitivities. Finally, the informational phase determines whether further information-gathering is profitable. At some point, further information will cost more than it is worth, and the alternative that currently has the highest certainty equivalent will be selected for implementation.

The iterative decision analysis described above is not intended to fit any particular situation exactly but, rather, all situations conceptually. A discussion follows on two procedures required to carry out the analysis: encoding knowledge and preferences.

**ENCODING KNOWLEDGE AND PREFERENCES**

Encoding Knowledge as Probability Distributions

Perhaps the single most unusual aspect of decision analysis is its treatment of uncertainty. Since uncertainty is the central problem in decision-making, it is essential to understand the conceptual and logical foundations of the approach to this issue.

The Importance of Uncertainty

The importance of uncertainty is revealed by the realization that decisions in situations where there is no random element can usually be made with little difficulty. Only when uncertainty exists about which outcome will occur is there a real decision problem.

For example, suppose that we are planning to take a trip tomorrow and that bad weather is forecast. We have the choice of flying or of taking a train. If a clairvoyant told us the consequences of each of these acts, then our decision would be very simple. Thus, if he said that the train would depart at 9:13 A.M. and arrive at 5:43 P.M. and if he described in detail the nature of the train accommodations, the dining car, and the people whom we would meet as traveling companions, then we would have a very clear idea of what taking the train implied. If he further specified that the plane would leave 2 hours late and arrive 2 1/2 hours late, stated that the flight would be especially bumpy during a certain portion of the trip, and described the meals that would be served and the acquaintances we would meet, then the flying alternative would be described as well.

Most of us would have little trouble in making a decision about our means of travel when we considered these carefully specified outcomes in terms of our tastes and desires. The decision problem is difficult because of the uncertainty of departure and arrival times and, in the case of the plane, even whether the trip would be possible at all. The factors of personal convenience and pleasure will be more or less important depending upon the urgency of the trip and, consequently, so will the uncertainties in these factors. Thus we cannot make a meaningful study of decision-making unless we understand how to deal with uncertainty. Of course, in the problems that are of major practical interest to the decision analyst, the treatment of uncertainty is even more pressing.

It is possible to show that the only consistent theory of uncertainty is the theory of probability invented 300 years ago and studied seriously by mathematicians the world over. This theory of probability is the only one that has the following important property: the likelihood of any event’s following the presentation of a sequence of points of data does not depend upon the order in which those data are presented. So fundamental is this property that many would use it as a defining basis for the theory.

The Subjective Interpretation of Probability

A reasonable question is: If probability is so essential to decision-making, why hasn’t
its importance been more widely appreciated until now? The answer is that many users of probability theory (but certainly not the original developers) considered probabilities to be physical parameters of objects, such as weight, volume, or hardness. For example, there was much mention of "fair" coins and "fair" dice, with the underlying notion that the probability of events associated with these objects could be measured in the real world.

For the past 15 years, however, an important minority of experts on the subject have been advancing the view that probabilities measure a person's state of knowledge about phenomena rather than the phenomena themselves. They would say, for example, that when someone describes a coin as "fair" he really means that on the basis of all evidence presented to him he has no reason for asserting that the coin is more likely to fall heads than tails. This view is modern, but not a product of modern times. It was studied clearly and convincingly 200 years ago but remained buried for a long time.

An example illustrating this view of probability follows: An astronaut is about to be fired into space on a globe-circling mission. As he is strapping himself into his capsule on top of a gleaming rocket, he asks the launch supervisor, "By the way, what's the reliability of this rocket?" The launch supervisor replies "Ninety nine percent—we expect only one rocket in one hundred to fail." The astronaut is reassured but still has some doubts about the success of his mission. He asks, "Are these rockets around the edge of the field the same type as the one I'm sitting on?" The supervisor replies, "They're identical." The astronaut suggests, "Let's shoot up a few just to give me some courage."

The rocket is fitted with a dummy payload, prepared for launching, and fired. It falls in the ocean, a complete failure. The supervisor comments, "Unlucky break, let's try another." Unfortunately, that one also fails by exploding in mid-air. A third is tried with disastrous results as it disintegrates on its pad. By this time, the astronaut has probably handed in his resignation and headed home. Nothing could convince him that the reliability of his rocket is still 99%.

But, in reality, what has changed? His rocket is physically unaffected by the failure of the other rockets. Its guidance system, rocket engine, and life support system are all exactly the same as they were before the other tests. If probability were a state of things, then the reliability of his rocket should still be 0.99. But, of course, it is not. After observing the failure of the first rocket, he might have evaluated the reliability of his rocket at, say, 0.90; after the second failure, at 0.70; and finally after the third failure, at perhaps 0.30. What happened was that his state of knowledge of his own rocket was influenced by what happened to its sister ships, and therefore his estimate of its reliability must decrease. His final view of its reliability is so low that he does not choose to risk his life.

The view of probability as a state of things is just not tenable. Probability should be considered as the reading of a kind of mental thermometer that measures uncertainty rather than temperature. The reading goes up if, as data accumulate, it tends to increase the likelihood of the event under consideration. The reading of 1 corresponds to certainty that the event will occur, the reading of 0 to certainty that it will not occur. The inferential theory of probability is concerned with the question of how the reading ought to fluctuate in the face of new data.

Encoding Experience

Most persons would agree that it would be unwise to make a decision without considering all available knowledge before acting. If someone were offered an opportunity to participate in a game of chance by his best friend, by a tramp, and by a business associate, he would generally have different feelings about the fairness of the game in each case. A major problem is how to encode the knowledge he has in a usable form. This problem is solved
by the observation that probability is the appropriate way to measure his uncertainty.

All prior experience must be used in assessing probabilities. The difficulty in encoding prior knowledge as probability is that the prior information available may range in form from a strong belief that results from many years of experience to a vague feeling that arises from a few haphazard observations. Yet there is probably not a person who had no information about an event that was important to him. People who start out saying that they have no idea about what is going to happen can always, when pressed, provide probability assignments that show considerable information about the event in question. The problem of those who would aid decision-makers is to make the process of assigning probabilities as simple, efficient, and accurate as possible.

The Practical Encoding of Knowledge

In the probabilistic phase of decision analysis, we face the problem of encoding the uncertainty in each of the aleatory variables. In organizational decision-making, prior probability distributions (or priors) should be assigned by the people within the organization who are most knowledgeable about each state variable. Thus, the priors on engineering variables will typically be assigned by the engineering department; on marketing variables, by the marketing department; and so on. However, since each case is an attempt to encode a probability distribution that reflects a state of mind and since most individuals have real difficulty in thinking about uncertainty, the method of extracting the priors is extremely important. As people participate in the prior-gathering process, their attitudes are indicated successively by: “This is ridiculous.” “It can’t be done.” “I have told you what you want to know, but it doesn’t mean anything.” “Yes, it seems to reflect the way I feel.” And “Why doesn’t everybody do this?” In gathering the information, the analyst must be careful to overcome the defenses the individual develops as a result of being asked for estimates that are often a combination of targets, wishful thinking, and expectations. The biggest difficulty is in conveying to the man that the analyst is interested in his state of knowledge and not in measuring him or setting a goal for him.

If the subject has some experience with probability, he often attempts to make all his priors look like normal distributions, a characteristic known as “bell-shaped” thinking. Although normal distributions are appropriate priors in some circumstances, they should not become foregone conclusions.

Experience has shown certain procedures to be effective in this almost psychoanalytic process of prior measurement. One procedure is to make the measurement in a private interview to eliminate group pressure and to overcome the vague notions that most people exhibit about probabilistic matters. Unless the subjects are already experienced in decision analysis, the distribution of forms on which they are supposed to draw their priors has proved worse than useless.

The interview begins with such questions as “What are the chances that x will exceed ten?” This approach is taken because people seem much more comfortable in assigning probabilities to events than they are in sketching a probability density function. The interviewer also skips around, asking the probability that x will be “greater than 50,” “less than ten,” “greater than 30,” often asking the same question again later in the interview. The replies are recorded out of the view of the subject so as to frustrate any attempt at forced consistency on his part. As the interview proceeds, the subject often considers the questions with greater and greater care, so that his answers toward the end of the interview may represent his feelings much better than did his initial answers.

The interviewer can change the form of the questions by asking the subject to divide the possible values of an aleatory variable into n intervals of equal probability. The answers to
all these questions enable the analyst to draw the excess probability distribution for the aleatory variable, a form of representation that seems easy to convey to people without formal probabilistic training.

The result of the interview must be a prior that the subject is willing to live with, regardless of whether it will describe a lottery on who buys coffee or on the disposal of his life savings. The analyst can test the prior by comparing it with known probabilistic mechanisms. For example, if the subject says that some aleatory variable $x$ is equally likely to be less or greater than $a$, then he should be indifferent about whether he is paid $100 if $x$ exceeds $a$ or if he can call the toss of a coin. If he is not indifferent, then he must change $a$ until he is. The end result of such questions is to produce a prior that the subject is not tempted to change in any way. Although the prior-gathering process is not cheap, the analyst need perform it only on the aleatory variables.

In cases where the interview procedure is not appropriate, the analyst can often obtain a satisfactory prior by drawing one himself and then letting the subject change it until the subject is satisfied. This technique may also be useful as an educational device in preparation for the interview.

If two or more aleatory variables are dependent, then the procedure requires priors that reflect the dependencies. The technique of prior gathering is generally the same but somewhat more involved. Since the treating of joint variables is a source of expense, the analyst should formulate the problem so as to avoid them whenever possible.

An Actual Probability Assessment

Figure 8 illustrates prior-gathering. The decision in a major problem was thought to depend primarily on the average lifetime of a new material. Since the material had never been made and test results would not be available until three years after the decision was required, it was necessary to encode how much knowledge the company now had concerning the life of the material. This knowledge resided in three professional metallurgists who were experts in that field of technology. These men were interviewed separately according to the principles described. They produced the points labeled "Subjects 1, 2, and 3" in the figure. These results have several interesting features. For example, for $t = 17$, Subject 2 assigned probabilities of 0.2 and 0.25 at various points in the interview. On the whole, however, the subjects were remarkably consistent in their assignments. Subject 3 was more pessimistic about the lifetime than was Subject 1.

Upon conclusion of the interviews, the three subjects were brought together, shown the results, and a vigorous discussion took place. Subjects 1 and 3 each brought forth information of which the other two members of the group were unaware. As the result of this information exchange, the three subjects drew the consensus curve—each said that this curve represented the state of information about the material’s life at the end of the meeting. Later, their supervisor said he understood their position on the new material for the first time.

It has been suggested that the proper way to reconcile divergent priors is to assign
weights to each, multiply, and add, but this experiment is convincing evidence that any such mechanistic procedure misses the point. Divergent priors are an excellent indicator of divergent states of information. The experience just described not only produced the company's present encoding of uncertainty about the material's lifetime, but at the same time encouraged and effected the exchange of information within the group.

Encoding New Information

Following the encoding of the original information about an aleatory variable by means of a prior probability distribution, or about an event by the assignment of a probability, the question naturally arises as to how these probability assignments should be changed in the light of new information. The answer to this question was provided by Bayes in 1763; it is most easily introduced by considering the case of an event. Suppose that we have assigned some probability \( p(A) \) to an event \( A \) occurring and that another event \( B \) is statistically related to \( A \). We describe this relationship by a conditional probability of \( B \) given \( A \), \( p(B|A) \), the probability of \( B \) if \( A \) occurs; assign this probability also. Now we are told that \( B \) has, in fact, occurred. How does this change the probability that \( A \) has occurred; in other words, what is the probability of \( A \) given \( B \), \( p(A|B) \)?

Bayes showed that to be logical in this situation, the probability of \( A \) given \( B \), \( p(A|B) \), must be proportional to the probability of \( A \), \( p(A) \), and the probability of \( B \) given \( A \), \( p(B|A) \). This relationship is expressed as \( p(A|B) \) is proportional to \( p(A) \) times \( p(B|A) \).

The important thing to remember is that any posterior (after new information) probability assignment to an event is proportional to the product of the prior probability assignment and the probability of the new information given that the event in question occurred. The same idea carries over in the much more complicated situations encountered in practice.

Thus, Bayes' interpretation shows how new information must be logically combined with original feelings. Subjective probability assignments are required both in describing the prior information and also in specifying how the new information is related to it. In fact, as already mentioned, Bayes' interpretation is the only method of data processing that ensures that the final state of information will be the same regardless of the order of data presentation.

Encoding Values and Preferences

The other subjective issue that arises in decision analysis is the encoding of values and preferences. It seems just as difficult to obtain an accurate measurement of desires as of information.

The value issue penetrates the core of the decision problem. Whether personal or organizational, the decision will ultimately depend on how values are assigned. If each alternative could produce only a single outcome, it would only be necessary to rank the outcomes in value and then choose the alternative whose outcome was highest in value. However, typically each alternative can produce many possible outcomes, outcomes that are distributed in time and also subject to uncertainty. Consequently, most real decision problems require numerical measures of value and of time and risk preference.

Measuring Value

The application of logic to any decision problem requires as one of its fundamental steps the construction of a value function, a scale of values that specifies the preference of the decision-maker for one outcome compared with another. We can think of the problem as analogous to the one we face if we have someone buy a car for us: We must tell our agent what features of the car are important to us and to what extent. How do we value performance relative to comfort, appearance relative to economy of operation, or other ratings?
To construct a value function in the car purchase problem, we can tell our agent the dollar value we assign to each component of a car's use. We might say, for example, that given our usage characteristics, a car that runs 18 miles to a gallon of gas is worth $40 a year more to us than a car that runs only 15 miles and that foam rubber seats are worth $50 more to us than ordinary seats. When we had similarly specified the dollar value of all the possible features of a car, including those whose values might not be additive, our agent would be able to go into the marketplace, determine the value and price of every offered car, and return with the most profitable car for us (which might, of course, be no car at all). In following this philosophy, we do not care if, in fact, there are any cars for sale that have all or any part of the features that we have valued. The establishment of the value function depends remotely, if at all, on the spectrum of cars available.

The main role of the value function is to serve as a framework of discussion for preferences. The value function encodes preferences consistently; it does not assign them. Consequently, the decision-maker or decision analyst can insert alternative value specifications to determine sensitivity of decisions to changes in value function. The process of assigning values will naturally be iterative, with components of value being added or eliminated as understanding of the problem grows.

A question that arises is, "Who should set the values?" In a corporate problem, to what extent do the values derive from management, stockholders, employees, customers, and the public? The process of constructing a value function brings into the open questions that have been avoided since the development of the corporate structure.

Establishing Time Preference

The general tendency of people and organizations is to value outcomes received sooner more highly than outcomes received later. In an organization, this phenomenon usually occurs in connection with a time stream of profit. Time streams that show a greater share of their returns in earlier time periods are generally preferred.

A number of concepts have arisen to cope with time preference in corporations. To illustrate these concepts, let \( x(n) \) be the cash flow in year \( n \) in the future, positive or negative, where \( n = 0 \) is the beginning of the present year, \( n = 1 \) next year, and so on. A positive cash flow indicates that income exceeds expenditures, a negative cash flow implies the reverse. Negative cash flows will usually occur in the early years of the project.

The most elementary approach, the payback period method, rests on the assumption that the cash flow will be negative in early periods and will then become and remain positive for the balance of the project. The payback period is the number of the period in which cumulative cash flow becomes positive.

The payback period came into common use when projects were typically investments in capital equipment, investments characterized by a high initial outlay gradually returned in the course of time. However, only a few modern investments have such a simple structure. The project may contain several interspersed periods of investment and return. There would seem to be little justification for use of the payback period in modern corporate decision-making.

The idea of internal rate of return was introduced as a more sophisticated time preference measure. The internal rate of return is derived from the present value of the project, defined by

\[
PV(i) = x(0) + x(1) \left( \frac{1}{1 + i} \right) + x(2) \left( \frac{1}{1 + i} \right)^2 + \cdots
\]

where \( i \) is interpreted as an annual interest rate for funds connected with the project. The internal rate of return is the value of \( i \) that makes the present value equal to zero; in
other words, the solution of the equation

\[ PV(i) = 0. \]

A justification offered for the use of internal rate of return is that application of the method to an investment that pays a fixed interest rate, like a bond or a bank deposit, produces an internal rate of return equal to the actual interest rate. Although this property is satisfying, it turns out to be insufficient justification for the method. One defect, for example, is that more than one interest rate may satisfy the equation; that is, it is possible for an investment to have two internal rates of return, such as 8% and 10%. In fact, it can have as many as the number of cash flows in the project minus one. A further criticism of the method is that it purports to provide a measure of the desirability of an investment that is independent of other opportunities and of the financial environment of the firm. Although meticulous use of internal rate of return methods can lead to appropriate time preference orderings, computing the present value of projects establishes the same ordering directly, without the disadvantages of internal rate of return. Furthermore, present value provides a measure of an investment such that the bigger the number, the better the investment. The question that arises is what interest rate \( i \) to use in the computation.

Much misunderstanding exists about the implications of choosing an interest rate. Some firms use interest rates like 20% or 25% in the belief that this will maintain profitability. Yet at the same time they find that they are actually investing most of their available capital in bank accounts. The overall earnings on capital investment will therefore be rather low. The general question of selecting \( i \) is too complicated to treat here, but the fundamental consideration is the relationship of the firm to its financial environment.

There is a cogent logical argument for the use of present value. If a decision-maker believes certain axioms regarding time streams—axioms that capture such human characteristics as greediness and impatience—then the time preference of the decision-maker for cash streams that are certain must be characterized by the present value corresponding to some interest rate. Furthermore, if a bank is willing to receive and disburse money at some interest rate, then, for consistency, the decision-maker must use this bank interest rate as his own interest rate in the calculation. Present value is therefore a well-founded criterion for time preference.

In this discussion of time preference, there has been no uncertainty in the value of cash streams. Undoubtedly, it was the existence of uncertainty that made payback periods and artificially high interest rate criteria seem more logical than they in fact are. Such procedures confuse the issues of time and risk preference by attempting to describe risk preference as a requirement for even greater rapidity of return. Decision analysis requires a clear distinction between the time and risk preference aspects of decision-making.

**Establishing Risk Preference**

The phenomenon of risk preference was discussed in connection with the proposition of tossing a coin, double or nothing, for next year’s salary: most people will not play. However, suppose they were offered some fraction of next year’s salary as an inducement to play. If this fraction is zero, there is no inducement, and they will refuse. If the fraction is one, they have nothing to lose by playing and they have a .5 probability of ending up with three times next year’s salary; clearly, only those with strange motivations would refuse. In experiments on groups of professional men, the fraction required to induce them to play varies from about 60% to 99%, depending on their financial obligations. Obviously, the foot-loose bachelor has a different attitude than does the married man with serious illness in the family.

The characteristic measured in this experiment is risk aversion. Few persons are indifferent to risk—i.e., willing to engage in a fair
gamble. Fewer still prefer risk—i.e., willing to engage in the kind of gambles that are unfair, such as those offered at professional gambling establishments. When considering sums that are significant with respect to their financial strength, most individuals and corporations are risk-averse.

A risk-averse decision-maker is willing to forego some expected value in order to be protected from the possibilities of poor outcomes. For example, a man buys life, accident, and liability insurance because he is risk-averse. These policies are unfair in the sense that they have a negative expected value computed as the difference between the premium and the expected loss. It is just this negative expected value that becomes the insurance company’s profit from operations. Customers are willing to pay for this service because of their extreme aversion to large losses.

A logical way to treat the problem of risk aversion is to begin with the idea of a lottery. A lottery is a technical term that refers to a set of prizes or prospects with probabilities attached. Thus, tossing a coin for next year’s salary is a lottery and so is buying a life insurance policy. The axioms that the decision-maker must satisfy to use the theory are:

- Given any two prizes in a lottery, he must be able to state which he prefers or whether he is indifferent between them. His preferences must be transitive: if he prefers prize A to B and prize B to C, he must also prefer A to C.
- If he prefers A to B and B to C, he must be indifferent to receiving B for certain or participating in a lottery with A and C as prizes for some probability of winning A.
- If he prefers A to B, then given the choice of two lotteries that both have prizes A and B, he will prefer the one with the higher probability of winning A.
- He treats as equivalent all lotteries with the same probabilities of achieving the same prizes, regardless of whether the prizes are won in one drawing, or as the result of several drawings that take place at the same time.

It is possible to show that an individual who wants to act in accordance with these axioms possesses a utility function that has two important properties. First, he can compute his utility for any lottery by computing the utility of each prize, multiplying by the probability of that prize, and then summing over all prizes. Second, if he prefers one lottery to another, then his utility for it will be higher.

If the prizes in a lottery are all measured in the same commodity, then, as discussed previously, the certain equivalent of the lottery is the amount of the commodity that has the same utility as the lottery. The concepts of utility and certain equivalent play a central role in understanding risk preference.

In the practical question of measuring risk preference, one approach is to present an individual with a lottery and ask him his certain equivalent. Or, we can provide the certain equivalent and all prizes but one and let him adjust the remaining prize until the certain equivalent is correct in his view. Finally, we can fix the certain equivalent and prizes and let him adjust the probabilities. All these questions permit us to establish the relationships between points on his utility curve and, ultimately, the curve itself. The interviewing in which the curve is measured is similar to that used for generating priors: the same need for education exists. The same types of inconsistency appear.

Although useful utility curves for individuals and organizations can be found in this manner, most decision-makers prefer to have some guidance in the selection of utility curves. The decision analyst can often provide this guidance by asking whether the decision-makers will accept additional axioms. One such axiom is: if all the prizes in the lottery are increased by some amount \( \Delta \), then the certain equivalent of the lottery will increase by \( \Delta \). The argument for the reasonableness of the axiom is very simple. The additional amount \( \Delta \) is money in the bank, no matter which prize in the lottery is won. Therefore, the new lottery should be worth
more than the original lottery. The counter argument is that having \( \Delta \) in the bank changes the psychological orientation to the original lottery.

If this \( \Delta \) axiom is added to the original set, then it is possible to show not just that a utility curve exists but that it must have a special form called the exponential form. A useful property of this exponential form is that it is described by a single number. This means that the analyst can characterize the utility curve of any individual or organization that wants to subscribe to these axioms by a single number—the risk aversion constant.

It is far easier to demonstrate to a decision-maker the consequences of his having different risk aversion coefficients and to measure his coefficient than it is to attempt to find a complete utility curve that is not of the exponential form. Encoding risk aversion in a single number permits measuring the sensitivity to risk aversion, as discussed earlier. In most practical problems, the entire question of risk aversion appears to be adequately treated by using the exponential form with a risk aversion constant appropriate to the decision-maker.

A cautionary note on the problem of practical measurement of risk aversion: experiments have revealed that the certain equivalents offered by subjects in hypothetical situations differ markedly from those offered when the situations are made real. This difficulty shows that the analyst must treat risk preference phenomena with great care.

**Joint Time and Risk Preference**

In most problems, both time and risk preference measures are necessary to establish the best alternative. Typically each outcome is represented by a time sequence of dependent uncertain values.

The question of how to describe preferences in such problems is fundamentally related to the way in which information on successive outcomes is revealed and to the extent to which it can help in making future decisions.

Two approaches illustrate the nature of the problem, each of which is appropriate under certain conditions. The first—that used in the original discussion of the probabilistic phase—is to compute the worth lottery implied by the model and then use the current utility function to develop the certain equivalent worth of the lottery. This approach is appropriate when there is no opportunity to utilize the information about outcomes as it is revealed, and thus where the prime interest is in the position occupied after all outcomes have been revealed.

Another approach is to imagine dealing with two agents. The first is a banker who will always pay immediately the amount specified by a particular company's time preference function applied to any time stream of values that is known with certainty. The other is a risk broker who will always pay the company's certain equivalent for any lottery. When faced with an uncertain stream of income, the company alternately deals with the risk broker to exchange lotteries for certain equivalents and with the banker to convert fixed future payments into present payments. The result of this alternating procedure is ultimately a single equivalent sum to represent the entire future process. Although appealing, the method may lead to the conclusion that the decision-maker should be willing to pay for "peace of mind" even when it has no effect on his financial future.

Thus the time-risk preference question ultimately depends on the decision-maker's tastes and options. The decision analyst can provide guidance in selecting from the many available approaches the one whose implications are best suited to the particular situation.

**APPLICATIONS**

In brief form, two examples illustrate the accomplishments and potential of decision analysis. In each case, the focus is on the key decision to be made and on the problems peculiar to the analysis.
New Product Introduction

A recent decision analysis was concerned with whether to develop and produce a new product. Although the actual problem was from another industry we shall suppose that it was concerned with aircraft. There were two major alternatives: to develop and sell a new aircraft \( A_2 \) or to continue manufacturing and selling the present product \( A_1 \). The decision was to be based on worth computed as the present value of future expected profits at a discount rate of 10% per year over a 22-year period. Initially, the decision was supposed to rest on the lifetime of the material for which the prior probability distribution, or priors, were obtained (Figure 8); however, a complete decision analysis was desired. Since several hundred million dollars in present value of profits were at stake, the decision analysis was well justified.

In the general scheme of the analysis, the first step was to construct a model for the business, as shown in Figure 9, which was primarily a model of the market. The profit associated with each alternative was described in terms of the price of the product, its operating costs, its capital costs, the behavior of competitors, and the natural characteristics of customers. Suspicion grew that this model did not adequately capture the regional nature of demand. Consequently, a new model was constructed that included the market character-

istics region by region and customer by customer. Moving to the more detailed basis affected the predictions so much that the additional refinement was clearly justified. However, other attempts at refinement did not affect the results sufficiently to justify a still more refined model.

Next, a sensitivity analysis was performed to determine the aleatory variables. These turned out to be operating cost, capital cost, and a few market parameters. Because of the complexity of the original business model, an approximation was constructed showing how worth depended on these aleatory variables in the area of interest. The coefficients of the approximate business model were established by runs on the complete model.

The market priors were directly assigned with little trouble. However, because the operating and the capital costs were the two most important in the problem, their priors were assigned according to a more detailed procedure. First, the operating cost was related to various physical features of the design by the engineering department; this relationship was called the operating cost function. One of the many input physical variables was the average lifetime of the material whose prior appears in Figure 8. All but two of the 12 physical input variables were independent. The priors on the whole set were gathered and used together with the operating cost function in a Monte Carlo simulation that produced a prior for the operating cost of the product.

The engineering department also developed the capital cost function, which was much simpler in form. The aleatory variables in this case were the production costs for various parts of the product. A simulation produced a prior on capital cost.

With priors established on all inputs to the approximate business model, numerical analysis determined the worth lottery for each alternative. The worth lotteries for the two alternatives closely resembled those in Figure 4, Part A. The new product alternative \( A_2 \) st
chastically dominated the alternative \( A_1 \) (continuing to manufacture the present product). The result showed two interesting aspects of the problem. First, it had been expected that the worth lottery for the new product alternative would be considerably broader than it was for the old product. The image was that of a profitable and risky new venture compared with a less profitable, but less risky, standard venture. In fact the results revealed that the uncertainties in profit were about the same for both alternatives, thus showing how initial impressions may be misleading.

Second, the average lifetime of the material whose priors appear in Figure 8 was actually of little consequence in the decision. It was true enough that profits were critically dependent on this lifetime if the design were fixed. But leaving the design flexible to accommodate to different average material lifetimes was not an expensive alternative. The flexible design reduced sensitivity to material lifetime so much that its uncertainty ceased to be a major concern.

The problem did not yield as easily as this, however. Figure 10 shows the present value of profits through each number of years \( t \) for each alternative. Note that if returns beyond year 7 are ignored, the old product has a higher present value; but in considering returns over the entire 22-year period, the relationship reverses. When managers saw these results they were considerably disturbed. The division in question had been under heavy pressure to show a profit in the near future, and alternative \( A_2 \) would not meet that requirement. Thus, the question of time preference that had been quickly passed off as one of present value at 10% per year became the central issue in the decision. The question was whether the division was interested in the quick kill or the long pull.

This problem clearly illustrates the use of decision analysis in clarifying the issues surrounding a decision. A decision that might have been made on the basis of a material lifetime was shown to depend more fundamentally on the question of time preference for profit. The extensive effort devoted to this analysis was considered well spent by the company, which is now interested in instituting decision analysis procedures at several organizational levels.

Space Program Planning

A more recent application in a quite different area concerned planning a major space program. The problem was to determine the sequence of designs of rockets and payloads that should be used to pursue the goal of exploring Mars. It was considered desirable to place orbiters about Mars as well as to land vehicles on the planet to collect scientific data.

The project manager had to define the design for each mission—that is, the type and number of launch vehicles, orbiters, and landers. The choice of design for the first mission could not logically be made without considering the overall project objectives and the feasible alternatives. Key features of the problem were the time for the development of new orbiting and landing vehicles, cost of each mission, and chances of achieving objectives.
Approach to Solution

To apply decision analysis to the problem posed, a two-phase program was adopted. The first or pilot phase consisted of defining a simplified version of the decision. To the maximum extent possible, however, the essential features of the problem were accurately represented and only the complexity was reduced. This smaller problem allowed easier development of the modeling approach, and exercising of the model provided insight into the level of detail required in structuring the inputs to the decision. The second phase consisted of developing the more realistic and complex model required to decide on an actual mission.

The Pilot Phase

To begin the decision analysis, four possible designs were postulated to represent increasing levels of sophistication. Figure 11 shows these designs and their potential accomplishments. The questions were: what design should be selected for the first opportunity, and what sequence of designs should be planned to follow the first choice? Should the project manager, for example, elect to provide the ultimate level of capability in the initial design in the face of uncertainties in the Martian environment and difficulties in developing complex equipment to survive the prelaunch sterilization environment? Or should he choose a much simpler design that could obtain some information about the Martian environment to be used in developing subsequent, more complex, vehicles.

Decision Trees

The heart of the model used in analyzing the decision was a decision tree that represented the structure of all possible sequences of decisions and outcomes and provided for cost, value, and probability inputs. Such trees contain two types of nodes (decision nodes and chance nodes) and two types of branches (alternative branches and outcome branches), as illustrated in Figure 12. Emanating from each decision node is a set of alternative branches, each branch representing one of the alternatives available for selection at that point of decision. Each chance node is fol-

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**Fig. 11—Configurations and Performance**

- **Example Configurations**
  - C1 Direct Entry Atmospheric Probe
  - C2 Orbital Entry Descent TV
  - C3 Orbital Entry Landed TV
  - C4 Orbital Entry-Life Detection Experiment

- **Example Outcome Levels**
  - L0 Current Achievement (Fly-by)
  - L1 Perform Atmospheric Experiments
  - L2 Return TV Pictures During Descent
  - L3 Return Landed TV Pictures and Perform Surface Property Experiments
  - L4 Perform Life Detection Experiments

**Fig. 12—Tree Relationships**

- **Branch Type:** Alternative, Outcome
- **Branch Parameters:** Costs, Values, Probabilities
- **Node Type:** Decision, Chance, Decision
- **Evaluated by:** Maximization, Expectation, Maximization
lowed by a set of outcome branches, one branch for each outcome that may be achieved following that chance node. Probabilities of occurrence and values are assigned to each of these outcomes; costs are assigned to each decision alternative.

Two fundamental operations, expectation and maximization, are used to determine the most economic decision from the tree. At each chance node, the expected profit is computed by summing the probabilities of each outcome, multiplied by the value of that outcome plus expected profit of the node following that outcome. At each decision node, the expected profit of each alternative is calculated as the expected profit of the following node ("successor node") less the cost of the alternative. The optimum decision is found by maximization of these values over the set of possible alternatives, i.e., by selecting the alternative of highest expected profit.

Order of Events

The particular sequence of mission decisions and outcomes was a significant feature of the pilot analysis. As illustrated in Figure 13, the initial event of significance was the selection of the 1973 mission configuration. However, since lead time considerations re-

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\[ S = \text{Select} \quad L = \text{Launch} \quad O = \text{Outcome} \]

quired that the 1975 configuration decision be made in 1972, the second mission decision had to be made prior to obtaining the first mission results. Similarly, the 1977 decision had to be made before obtaining the results of the 1975 mission, although after the 1973 mission results. In general, then, a mission configuration was made in ignorance of the results of the previous mission.

Tree Example

A complete decision tree for the pilot project, with the additional assumption that L2 is the highest level of success, is presented in Figure 14. The model that produces the numerical probabilities, values, and costs used in the example will be discussed later. Node 1, at the left side of the tree is the initial decision to select either a C1 or a C2 for the first launch opportunity. The box designated LO above this node indicates that the state at this node is the current level of achievement. Suppose a C1 is selected. The cost of that C1 is $850 million, indicated by the "-850" that is written under that branch. As a result of this choice, the next node is decision node 2. The box designated LO, C1 above this node indicates that the state of this node is the current level of achievement and a C1 is being constructed for the first launch. Now either a C1 or C2 must be selected for the second launch. If a C1 is selected, the cost is $575 million, and the next node is chance node 7. The two branches following this node represent the possible outcomes of the first launch. The LO' outcome which would be failure to better LO on the first try, occurs with probability 0.1 whereas the L1 outcome occurs with probability 0.9. The value of the LO' outcome is zero, whereas the value of the LO outcome is 1224. Now follow the case of the L1 outcome to decision node 34. The state L1, C1 at this node, means that the highest level of success is L1 and that a C1 is being constructed for the next launch. Since L1 has already been achieved at this point in the tree, a C2 is the only design that may be launched in the third opportunity, at a cost of $740 million. This leads to decision node 35, where the state is L1, C2.

Node 35 in the example tree illustrates coalescence of nodes, a feature vital to maintaining a manageable tree size. Node 35 on the upper path through the tree can be reached from four other paths through the tree as in-
dicated in the exhibit. If the coalescence did not occur, the portion of the tree following node 35 would have to be repeated four additional times. In the full pilot tree, coalescence results in a reduction of the number of branches in the tree by a factor of 30.

Along the path 1-2-7-34-35, at decision node 35, a C2 must be selected for the fourth opportunity. At chance node 36, the outcome of the third launch is either an L1′ (failure to better L1 with one attempt, which leads to node 38), or an L2 (which achieves a value of 1714 and successfully completes the program). These outcomes occur with probability 0.3 and 0.7, respectively. If L1′ is the outcome, chance node 38 is reached, where the outcome of the fourth launch is represented. The probability of L1″ is 0.24, and the probability of L2 is 0.76. Note that the probability of 12 has increased over that of node 36 (0.7 to 0.76) because of the experience gained previously.

One can similarly follow and interpret many other paths through the tree. A policy is a complete selection of particular alternatives at all decision nodes. This limits the set of all possible paths to a smaller subset. (It is not possible, for example, to reach node 26 if a C1 is chosen at node 1.) The probabilities, values, and cost of these paths then determine the characteristics of the decision policy.

The most economic policy, given the input data specifications, is defined as the policy that maximizes the expected profit of the project, i.e., expected value less expected cost.

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**Fig. 14**
Example of a Decision Tree

- **LO, C1**: Decision Node
- **LO, C2**: Chance Node
- **LO, C3**: Terminal Node-Success
- **LO, C4**: Terminal Node-Failure
- **LO, C5**: Node Number
- **LO, C6**: “State” of project at node
- **941**: Expected profit at given node
- **1224**: Cash flow (cost or value) occurring if branch is taken
- **.60**: Probability that branch will be taken
The technique illustrated here eliminates many of the nonoptimum policies from explicit consideration; it is the "roll back" technique that starts from the right side of the tree and progresses left to the beginning of the tree, making all decisions and calculations in reverse chronological order. Thus, when each decision is made, only policies that optimize decisions for the following decision nodes are considered.

Consider node 38 in Figure 14. At this chance node the probability of achieving $L_1''$, which is worth nothing, is 0.24, and the probability of achieving $L_2$, which is worth 1714, is 0.76. Thus, the expected profit of node 38 is: $0.24(0) + 0.76(1714) = 1303$. This number is written near node 38.

The calculations are carried out in this manner backwards through the tree. The first decision node with more than one choice is node 2. If a $C_1$ is selected, it costs $575$ million ($-575$) and leads to node 7 with an expected profit of 1408, which yields $-575 + 1408 = 833$. If a $C_2$ is selected, it costs $740$ million ($-740$) and leads to node 12 with an expected profit of 2106, which yields $-740 + 2106 = 1366$. Since 1366 is greater than 833, the most economic decision is to select a $C_2$ at node 2, which results in an expected profit of 1366.

Finally, the first decision is a choice between a $C_1$ with an expected profit of 516 or a $C_2$ with an expected profit of 832. Maximum expected profit is achieved by the choice of a $C_2$ resulting in an expected profit of 832. This
is the expected profit of the entire project at the time the first decision is made.

Figure 15 illustrates the complexity of the completed decision tree for the pilot phase of the analysis.

**Value Assignment**

A particularly important part of this study was the specification of the value to be attached to the outcomes of the program. Since the decision-makers were reluctant to state values in dollar terms, a tree of point values was employed. The value tree is simply a convenient way of showing how the total value of the project is to be broken down into its component outcomes. Figure 16 shows a value tree for the pilot analysis. The points assigned to each tip of the tree are the fraction of total program value assigned to this accomplish-

ament; the values accumulate as the program progresses. A total dollar value assigned to a perfect program therefore determines the dollar values used in the decision tree.

To derive a value measure, a value tree is constructed by considering first the major components of value and then the subcategories of each type, which are identified in more and more detail until no further distinction is necessary. Then each tip of the tree (constructed as above) is subdivided into four categories, each corresponding to the contribution of one of the four levels of achievement within the value subcategory represented by that tip.

The number 1.0 attached to the node at the extreme left of the value tree for the pilot analysis represents the total value of all the objectives of the pilot project (thus, the value of achieving L1, L2, L3, and L4). The four branches emanating from this node represent the four major categories of value recognized by the pilot model. The figure 0.62 attached to the upper branch represents the fraction of total value assigned to science. Two branches emanate from the science node, and 60% of the science value falls into the category of biological science. The 0.37 attached to the biological science node represents the fraction of total value attached to biological science, and is obtained by taking 60% of 0.62 (the fraction of total value attached to all science). Finally, the bottom branch following the biological science node indicates that 78% of the biological science value is achieved by jumping from L3 to L4.

The final step in value modeling is to obtain the fraction of total value to be attached to achieving each of the four levels. If all the contributions to achieving L1 (e.g., contributions to world opinion, U.S. public favor, physical science) are added, the result is the fraction of value that should be attached to achieving L1. The same process is followed for reaching L2 from L1, L3 from L2, and L4 from L3. The results of such a calculation are presented in the lower left corner of Figure 16.
Summary

On the basis of the promising results of working with the pilot model, a more complete model was developed to encompass nearly all of the factors involved in selecting the actual mission. It provided a more precise structure for assigning initial values, probabilities, and costs, and for updating probabilities and costs based on results achieved. The following tabulation shows a summary comparison of the complexity of the pilot model with the more complete model.

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<th>Pilot</th>
<th>Feature</th>
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<tr>
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<td>Mission Designs</td>
<td>14</td>
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<td>5</td>
<td>Outcomes</td>
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<td>56</td>
<td>Decision Tree Nodes</td>
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<td>1592</td>
<td>Paths Through Tree</td>
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Clearly, the full-scale decision tree could not be represented graphically. The tree was constructed and evaluated by computer programs specially developed for this application.

A model such as the one described here can be a valuable tool throughout the life of a project. As the project progresses, the knowledge of costs, probabilities, and values will improve as a result of development programs and flights. Improved knowledge can be used in the decision process each time a design must be selected for the next opportunity.

An important additional benefit of this analysis is that it provides a language for communicating the structure of the space project and the data factors relevant to the project decisions. It provides a valuable mechanism for discourse and interchange of information, as well as a means of delegating the responsibility for determining these factors.

FUTURE TRENDS

Decision analysis should show major growth, both in its scope of applications and in its effect on organizational procedures. This section presents various speculations about the future.

Applications

Market Strategy Planning

The importance of decision-making in a competitive environment has stimulated the use of decision analysis in both strategic and tactical marketing planning. The strategic problems are typically more significant because they affect the operations of the enterprise over many years. Strategic analysis entails building models of the company and of its competitors and customers, analyzing their interactions, and selecting strategies that will fare well in the face of competitive activities. Since most of this work is of a highly confidential nature, little has appeared in the public literature; nevertheless, there is reason to believe that many large U.S. corporations are performing work of this kind, however rudimentary it may be. The competitive analyses of a few quite sophisticated companies might rival those conducted in military circles.

Resource Exploration and Development

Resource exploration by mineral industries is a most natural application for decision analysis. Here the uncertainty is high, costs are great, and the potential benefits extremely handsome. At all levels of exploration—from conducting aerial surveys, through obtaining options on drill-test locations, to bidding and site development—decision analysis can make an important contribution. Organizations approaching these problems on a logical, quantitative basis should attain a major competitive advantage.

Capital Budgeting

In a sense, all strategic decision problems of a corporation are capital budgeting problems, for its ultimate success depends upon how it allocates its resources. Decision analysis should play an increasingly important role in
the selection of projects and in objective comparisons among them. Problems in spending for research and development programs, investment in new facilities, and acquisitions of other businesses will all receive the logical scrutiny of decision analysis. The methodology for treating these problems already exists; it now remains for it to be appreciated and implemented.

Portfolio Management

The quantitative treatment of portfolio management has already begun but it will receive even more formal treatment in the hands of decision analysts. The desires of the investing individual or organization will be measured quantitatively rather than qualitatively. Information on each alternative investment will be encoded numerically so that the effect of adding each to the portfolio can be determined immediately in terms of the expressed desires. The human will perform the tasks for which he is uniquely qualified: providing information and desires. The formal system will complement these by applying rapid logic.

Social Planning

On the frontiers of decision analysis are the problems of social planning. Difficult as it may be to specify the values and the criteria of the business organization, this problem is minor compared with those encountered in the public arena. Yet if decision-making in the public sector is to be logical, there is no alternative.

The problems to which a contribution can be made even at the current stage of development are virtually endless: in decisions associated with park systems, farm subsidies, transportation facilities, educational policy, taxation, defense, medical care, and foreign aid, the question of values is central in every case.

The time may come when every major public decision is accompanied by a decision analysis on public record, where the executive branch makes the decision using values specified by the people through the legislative branch. The breakdown of a public decision problem into its elements can only serve to focus appropriate concern on the issues that are crucial. For the first time, the public interest could be placed “on file” and proposals measured against it. A democracy governed in this fashion is probably not near at hand, but the idea is most intriguing.

Procedures

The effect of decision analysis on organizational procedures should be as impressive as its new applications. Some of the changes will be obvious, others quite subtle.

Application Procedures

Standardization by type of application will produce special forms of analyses for various types of decisions—for example, marketing strategy, new product introduction, research expenditures. This standardization will mean special computer programs, terminology, and specialization of concepts for each application. It will also mean that the important classes of decisions will receive much more effective attention than they do now.

Analytical Procedures

Certain techniques, such as deterministic, stochastic, and economic sensitivity analyses that may be performed with the same logic regardless of the application will be carried out by general computer programs. In fact, the process of development is well under way at the present time. Soon the logical structure of any decision analysis might be assembled from standard components.

Probabilistic Reporting

The introduction of decision analysis should have a major impact on the way organizational reporting is performed externally and internally. Externally, the organization will be able to illustrate its performance not just historically by means of balance sheets and operating statements, but also projectively by
showing management's probability distributions on future value. Since these projections would be the result of a decision analysis, each component could be reviewed by interested parties and modified by them for their own purposes. However, management would have a profitable new tool to justify investments whose payoffs lie far in the future.

Organizational management will acquire new and more effective information systems as a result of decision analysis. Internal reporting will emphasize the encoding of knowledge in quantitative form. Instead of sales forecasts for next year, there will be probability distributions of sales. Thus, the state of information about future events will be clearly distinguished from performance goals.

**Delegation by Value Function**

An important logical consequence of decision analysis is that delegation of a decision requires only transmission of the delegator's present state of information and desires. Since both of these quantities can be made explicit through decision analysis, there should be an increase in the extent and success of delegation. In the external relationships of the firm, the delegation will no doubt appear as an increased emphasis on incentive contracts, where the incentives reflect the value function of the organization to the contractor. This trend is already evident in defense contracting.

Internally, the use of the value function for delegation should facilitate better coordination of the units of the organization. If explicit and consistent values are placed on the outcomes of production, sales, and engineering departments, then the firm can be sure that decisions in each unit are being made consistently with the best overall interests of the firm. The goal is to surround each component of the organization with a value structure on its outputs that encourages it to make decisions as would the chief decision-maker of the organization if he were closely acquainted with the operations of the component.

**Organizational Changes and Management Development**

The introduction of decision analysis will cause changes in organizational behavior and structure. A change should take place in the language of management, for the concepts discussed in this report are so relevant to the decision-making process that, once experienced in using them, it is difficult to think in any other terms. The explicit recognition of uncertainty and value questions in management discussions will in itself do much to improve the decision-making process.

Special corporate staffs concerned with the performance of decision analysis are already beginning to appear. These people would be specially trained in decision analysis, probability, economics, modeling, and computer implementation. They would be responsible for ensuring that the highest professional standards of logic and ethics are observed in any decision analysis.

Special training for decision analysts will be accompanied by special training for managers. They will need to know much more than they do now about logical structure and probability if they are to obtain full advantage from the decision analyst and his tools. No doubt much of this training will occur in special courses devoted to introducing decision analysis to management. These courses will be similar to, but more fundamental than, the courses that accompanied the introduction of computers into the U.S. economy.

**Management Reward**

Encouraging managers to be consistent with organizational objectives in decision-making requires adjusting the basis for their rewards to that objective. If rewarded only for short run outcomes, they will have no incentive to undertake the long range projects that may be in the best interest of the organization. It follows that any incentive structure for management will have to reward the qual-
ity of decisions rather than the quality of outcomes. The new financial statements that show probability distributions on future profit would be the key to the reward structure. After these distributions had been "audited" for realism, the manager would receive a reward based upon them in a predetermined way. Thus, the manager who created many new investment opportunities for a company could be rewarded for his efforts even before any were fully realized.

To make this system feasible requires distinguishing between two kinds of managers: the one who looks to the future and prepares for it; and the one who makes sure that today's operations are effective and profitable. The distinction is that between an admiral and a captain, or between the general staff and the field commanders. Specialization of function in corporate management with significant rewards and prestige attached to both planning and execution could be the most important benefit of decision analysis.

**CONCLUSION**

Although an organization can achieve ultimate success only by enjoying favorable outcomes, it can control only the quality of its decisions. Decision analysis is the most powerful tool yet discovered for ensuring the quality of the decision-making process: its ultimate limit is the desire of the decision-maker to be rational.
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